1. The trisection points, A & B, can be computed by $\vec{A} = \vec{P} + \frac{1}{3}\vec{PQ}$ & $\vec{B} = \vec{P} + \frac{2}{3}\vec{PQ}$, or equivalently using $\vec{Q} + t\,\vec{QP}$, $t = 1/3$ & $t = 2/3$.

$\vec{P} + t\,\vec{PQ} = (8\vec{i} + 5\vec{j}) + t(-7\vec{i} - 12\vec{j})$

$t = 1/3$: $\vec{A} = (8 + \frac{-7}{3})\vec{i} + (5 + \frac{-12}{3})\vec{j} = \frac{17}{3}\vec{i} + \vec{j}$, so $\boxed{A(\frac{17}{3}, 1)}$

$t = 2/3$: $\vec{B} = (8 - \frac{14}{3})\vec{i} + (5 + \frac{-24}{3})\vec{j} = \frac{10}{3}\vec{i} - 3\vec{j}$, so $\boxed{B(\frac{10}{3}, -3)}$

2. $\vec{W} = 12 \cdot \frac{\vec{v}}{|\vec{v}|} = 12 \cdot \frac{(\vec{i} + 2\vec{j} - 2\vec{k})}{\sqrt{1^2+2^2+2^2}} = \boxed{\frac{12\sqrt{5}}{5}\vec{i} + \frac{24\sqrt{5}}{5}\vec{j} - \frac{24\sqrt{5}}{5}\vec{k}}$

3a) $\vec{A} \cdot \vec{B} = 1 \cdot 3 + 10 \cdot 0 + (-2)(-4) = \boxed{11}$

3b) To find the angle θ between $\vec{A}$ & $\vec{B}$, solve for θ:

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta = \sqrt{1^2+10^2+2^2} \cdot \sqrt{3^2+4^2}\cos\theta = 11$ (part a)

so $\cos\theta = \frac{11}{\sqrt{105} \cdot 5} \Rightarrow \theta = \cos^{-1}\left(\frac{11}{\sqrt{105}(5)}\right) \approx \boxed{77.602^\circ}$

3c) Vector projection of $\vec{B}$ on $\vec{A}$: $\left(\frac{\vec{B} \cdot \vec{A}}{|\vec{A}|^2}\right)\vec{A} = \frac{11}{105}(\vec{i} + 10\vec{j} - 2\vec{k})$

$= \frac{11}{105}\vec{i} + \frac{110}{105}\vec{j} - \frac{22}{105}\vec{k} = \boxed{\frac{11}{105}\vec{i} + \frac{22}{21}\vec{j} - \frac{22}{105}\vec{k}}$

4. $2x + 3y + 4z + D = 0$, solved by $(x, y, z) = (1, 2, -3)$

$\Rightarrow 2 + 6 - 12 + D = 0 \Rightarrow D = 4$: $\boxed{2x + 3y + 4z + 4 = 0}$

5. $A(2, -2, -1)$, $B(-3, 4, 1)$, $C(4, 2, 3)$. A normal vector to the plane containing A, B, C can be found various ways. One is:

$$\vec{AB} \times \vec{AC} = \vec{N} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & 6 & 2 \\ 2 & 4 & 4 \end{vmatrix} = 16\vec{i} + 24\vec{j} - 32\vec{k}$$

more convenient is $\frac{1}{8}\vec{N} = 2\vec{i} + 3\vec{j} - 4\vec{k}$, (same direction)

The plane: $2x + 3y - 4z + D = 0$, solve for D using A or B or C.

using A: $2(2) + 3(-2) - 4(-1) + D = 0 \Rightarrow 2 + D = 0 \Rightarrow D = -2$

plane: $\boxed{2x + 3y - 4z - 2 = 0}$ (or any non-zero multiple of this equation or equivalent equation)

(6.) Find the angle between the normal vectors to the 2 planes : $\vec{N_1} = 2\vec{i} + \vec{j} + \vec{k}$, $\vec{N_2} = 2\vec{i} - 2\vec{j} + \vec{k}$
If it is acute... done, else use the suppliment of the angle if it is obtuse.

θ: $|\vec{N_1}|\,|\vec{N_2}| \cos\theta = (2)(2) + (1)(-2) + (1)(1)$

$\Rightarrow \sqrt{6}\,(3)\cos\theta = 3$, $\cos\theta = \frac{3}{3\sqrt{6}} = \frac{\sqrt{6}}{6}$

$\theta = \cos^{-1}\left(\frac{\sqrt{6}}{6}\right) \approx \boxed{65.905^{\circ}}$

(7) $d = \dfrac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$, $(x_1, y_1, z_1) = (0, 1, 3)$

$Ax + By + Cz + D = 0 \longleftrightarrow 2x - y + 2z + 3 = 0$

$d = \dfrac{|(2)(0) + (-1)(1) + (2)(3) + 3|}{\sqrt{2^2 + 1^2 + 2^2}} = \boxed{\dfrac{8}{3}}$

(8a) $(x_1, y_1, z_1) = (4, -3, 5)$, $\vec{N} = A\vec{i} + B\vec{j} + C\vec{k}$
$= -2\vec{i} + 3\vec{j} + 4\vec{k}$

symmetric eqns: $\dfrac{x - x_1}{A} = \dfrac{y - y_1}{B} = \dfrac{z - z_1}{C}$

$\rightarrow \boxed{\dfrac{x-4}{-2} = \dfrac{y+3}{3} = \dfrac{z-5}{4}}$

(8b) Parametric equations : set each expression in (8a) equal to t and solve for the variable:

$\dfrac{x-4}{-2} = t \longrightarrow x = -2t + 4$

$\dfrac{y+3}{3} = t \longrightarrow y = 3t - 3$

$\dfrac{z-5}{4} = t \longrightarrow z = 4t + 5$

$t \in \mathbb{R}$.

(9) $\vec{PQ} = 4\vec{i} + \vec{j} + 8\vec{k} = A\vec{i} + B\vec{j} + C\vec{k}$

$|\vec{PQ}| = \sqrt{4^2 + 1^2 + 8^2} = 9 = d$

$\cos\alpha = \dfrac{A}{d} = \dfrac{4}{9}$

$\cos\beta = \dfrac{B}{d} = \dfrac{1}{9}$

$\cos\gamma = \dfrac{C}{d} = \dfrac{8}{9}$

$$\boxed{\begin{aligned} \alpha &= \cos^{-1}\left(\tfrac{4}{9}\right) \approx 63.612^{\circ} \\ \beta &= \cos^{-1}\left(\tfrac{1}{9}\right) \approx 83.621^{\circ} \\ \gamma &= \cos^{-1}\left(\tfrac{8}{9}\right) \approx 27.266^{\circ} \end{aligned}}$$

(10) $\vec{C} = \vec{A} \times \vec{B}$ is defined by

$\boxed{a}$ $\vec{C}$ normal to the plane containing $\vec{B}$ & $\vec{A}$ (equiv. $\vec{C}$ is perpendicular to both $\vec{A}$ & $\vec{B}$)

$\boxed{b}$ $|\vec{C}| = |\vec{A} \times \vec{B}| = |\vec{A}|\,|\vec{B}| \sin\theta$, θ the angle between $\vec{A}$ & $\vec{B}$.

$\boxed{c}$ the "right-hand" orientation. (a) leaves open 2 directions. The right-hand fingers curling from $\vec{A}$ towards $\vec{B}$ will result in the extended thumb to point in the direction of $\vec{C}$.

(11) $\vec{A} = \vec{i} + 10\vec{j} - 2\vec{k}$, $\vec{B} = 3\vec{i} + 0\vec{j} - 4\vec{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 10 & -2 \\ 3 & 0 & -4 \end{vmatrix} = \boxed{-40\vec{i} - 2\vec{j} - 30\vec{k}}$$

(12) $A(1,2,3)$ $B(2,12,1)$, $C(4,2,-1)$ vertices of ΔABC

If $\vec{V} = \vec{AB} \times \vec{AC}$ then $|\vec{V}|$ is the area of the parallelogram made by 2 copies of ΔABC, so area $\Delta ABC = \frac{1}{2}|\vec{V}|$

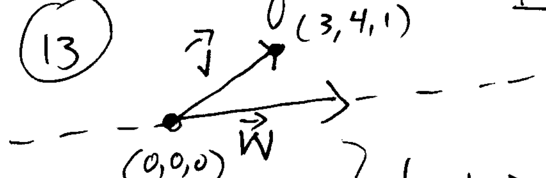
$$\vec{V} = (\vec{i} + 10\vec{j} - 2\vec{k}) \times (3\vec{i} + 0\vec{j} - 4\vec{k})$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 10 & -2 \\ 3 & 0 & -4 \end{vmatrix} = -40\vec{i} - 2\vec{j} - 30\vec{k}$$

$$|\vec{V}| = \sqrt{40^2 + 2^2 + 30^2} = \sqrt{2504} = 2\sqrt{626}$$

area of $\Delta ABC = \boxed{\sqrt{626}}$

(13)



$\frac{x}{2} = \frac{y}{3} = \frac{z}{1} \Rightarrow (0,0,0)$ is a point on the line.

& $2\vec{i} + 3\vec{j} + \vec{k} = \vec{W}$

$\vec{u}$ = unit vector in direction of $\vec{W}$

$$= \frac{\vec{W}}{\sqrt{2^2+3^2+1^2}} = \frac{\vec{W}}{\sqrt{14}} = \frac{1}{\sqrt{14}}(2\vec{i} + 3\vec{j} + \vec{k})$$

$v = 3\vec{i} + 4\vec{j} + \vec{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{vmatrix} \cdot \frac{1}{\sqrt{14}}$$

$d = |\vec{u} \times \vec{v}|$

$$= (-\vec{i} + \vec{j} - \vec{k})\frac{1}{\sqrt{14}}, \quad |\vec{u} \times \vec{v}| = \frac{\sqrt{3}}{\sqrt{14}} = \frac{\sqrt{42}}{14}$$

(14) $d = |\vec{P_1P_2} \cdot \vec{u}|$, $\vec{u}$ a unit vector parallel to $\vec{v}_1 \times \vec{v}_2$, $P_1(2,1,-1)$, $P_2(4,0,5)$

$$\vec{V}_1 \times \vec{V}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 2 & 3 \\ -1 & 4 & -5 \end{vmatrix} = -22\vec{i} - 18\vec{j} - 10\vec{k}, \quad \vec{u} = \frac{-22\vec{i} - 18\vec{j} - 10\vec{k}}{\sqrt{908}}$$

$\vec{P_1P_2} = 2\vec{i} - \vec{j} + 6\vec{k}$

$$|\vec{u} \cdot \vec{P_1P_2}| = \frac{86}{\sqrt{908}} = \cancel{\frac{86\sqrt{908}}{908}}$$

$$\cancel{\frac{43\sqrt{227}}{227}}$$